SMS EULER 300TH ANNIVERSARY ESSAY COMPETITION

LEONHARD EULER

Submitted by S2 Students

Tan Li Hong
Yap Nian Fang Melody
Joann Ong Si Lin

Paya Lebar Methodist Girls' School (Secondary)

2007

ABSTRACT

Leonhard Euler was a Swiss mathematician and physicist. Euler was considered to be the preeminent mathematician on the early 18th century and one of the greatest of all time. Leonhard Euler was considered to be a genius and came up with several formulas and theorems.

In this report, we would be concentrating on two of Euler's formulas. This report and project aims to help people in knowing more about the background of these formulas and the founder of the formulas.

1. INTRODUCTION & BACKGROUND

Leonhard Euler has produced more than 380 works on topics such as calculus of variations, the calculation of orbits, ballistics, analysis, lunar motion and differential calculus, most of these were produced during the 1740s when he accepted to work at the Berlin Society of the Sciences where he stayed for 25 years as the director of the society, which was largely transformed by him. In 1727 Euler joined the Academy of Science in St. Petersburg, Russia where he became a professor of physics in 1730 and professor of mathematics in 1733. Euler has also received 12 prizes from the Paris Académie des Sciences from 1738 to 1772.

Leonhard Euler was born in Basel, Switzerland on 15 April 1707. He went to Russia in 1727 and to Berlin in 1741. When Euler eventually returned to St. Petersburg in 1766, a brief illness left him completely blind which hampered his ability to do research, but he was able to continue his work with the help of his assistants whom he dictated his thoughts to. Half his total works were produced after 1765 and though he was completely blind, his articles became more concise. His memory remained flawless and he continued to have original ideas.

Euler’s activity in the academy was unabated when he died of a stroke on 18 September 1783. Euler was one of the most important mathematicians since Sir Isaac Newton. He was deeply interested in the applications of mathematics, but would develop the pertinent mathematics to deep levels of abstraction and generality.

Many modern conventions and notations are due to him; such as the symbol (x) for the value of a function and i for the square root of -1. In number theory, Euler was concerned with the theory of divisibility, introducing the so-called Euler's function, which tallies the quantity of divisors of a given integer. These led him to the discovery of the law of quadratic reciprocity, whose complete proof was later established by Carl Friendrich Gauss. His methods were algebraic, but Euler was the first to introduce analytic methods into number theory. Euler studied mathematical constants, such as *e* and *pi*, as well as Euler's constants. Euler stated the theorem that on algebraic polynomial of degree *n* roots of the form *a1bi*, which is now known as the fundamental theorem of algebra.

Euler also attempted to derive an exact formula for the roots of the 5th degree polynomial and his failures led him to use approximation methods of numerical analysis. Although many mathematicians had studied infinite series, Euler was unusually successful in the calculation, deriving simple formulas for the sums of reciprocal of even powers of integers. Through these series, he studied special functions and discovered Euler's constant for the approximation of the harmonic series. Euler presented the idea that mathematical analysis is the study of functions; he clearly defined the concept of a function, which closely approximates the modern notion. He advanced the knowledge of complex numbers, discovering the differential equation that relate the real and imaginary partsot an analytic function, Euler applied his techniques to the computation of the real integrals.

Euler made great contributions to the field of differential equations, including the method of variation of constants as well as the use of characteristics curves. Some of the applications of this work involve vibrating string problems, hydrodynamics and the motion of air in pipes. His studies in the calculus of variations led him to the

Eulerian differential equation, and his exposition of the subject became classical. Euler was the first to formulate the principal problems of this subject and the main methods of solution. He was also the first author in topology, solving the famous riddle of seven bridges of königsberg; he studied polyhedra, deriving what later became known as the Euler characteristic – a formula relating to the number of edges, faces and vertices.

Besides these contributions to pure Math, Euler laboured in mechanics, astronomy and optics. Euler was a humble man, but also one of the greatest scientists and mathematicians of all time, regarded by his peers as an astounding genius. His mathematical research has stimulated an enormous amount of subsequent activity, and many of his ideas were ahead of their time.

2. REVIEW

Leonhard Euler was a Swiss mathematician and a physicist, who spent most of his life in Russia and Germany. Euler made important discoveries in fields as diverse as calculus and topology. Euler also introduced much of the modern mathematical terminology and notation particularly for mathematical analysis, such as the notation of a mathematical function.

Leonhard Euler was born in Basel on 15 April 1707. He entered the University of Basel when he was only 14. In the next five years, he received the Bachelor of Arts degree and masters in physiology. At 16, he joined the theology department.

2.1 EULER'S NUMBER

The base of natural logarithms is a mathematical constant *e*, also known as Euler's number; *e* is approximately equals to 2.71828(to 5 decimal places). The base *e* is derived from adding up a few terms of this series

$$e = 1 + 1/1! + 1/2! + 1/3! + 1/4! + 1/5! + 1/6! + 1/7! \ldots$$

This decimal expansion is irrational, as it never ends and the numbers never repeat.

2.2 EULER'S FORMULA (I)

He worked on the mathematics behind the Greek concept of "perfect bodies". The Greeks defined the perfect body as the form built from identical regular polyhedral. There are only 5 such bodies namely polyhedral (more than three sides), tetrahedron (4 triangular faces), cube, octahedron (8 triangular faces), dodecahedron (10 regular pentagonal faces) and icosahedrons (20 triangular faces). The vertices (V), edges (E) and faces (F) of a polyhedron can be thought of as a connected graph, e.g. cube with its top face removed and pushed flat onto a plane yields a graph with 8 vertices and 12 edges dividing the plane into regions. Euler was able to relate the number of faces, vertices and edges of a polyhedron by the following equation:

$$F+V= E+2$$

from which one can derive that there are only 5 regular polyhedrons.

2.3 EULER'S FORMULA (II)

The formula $e^{ix} = \cos x + i\sin x$ is today known as Euler's formula. It has some interesting conseguences: setting $x= \pi$, we obtain:

$$e^{i\pi} = cos\ \pi + isin\pi = -1 + i.0 + -1$$

Mathematicians often deem this as one of the most beautiful facts of mathematics: it is a remarkably simple equation that connects the mysterious and persuasive numbers *e, π and -1.* Euler's formula provides a very simple means for deriving (and memorizing) certain identities from trigonometry for example, since

$$e^{iA} \times e^{iB} = e^{i(A+B)}$$

we have: $(\cos A + i\sin A) . (\cos B + i\sin B) = \cos(A + B) + i\sin(A + B)$

Expanding the brackets on the left and collecting terms that contain *i* and those that do not yields:

$$\cos(A + B) = \cos A . \cos B - \sin A . \sin B$$

$$\sin(A + B) = \sin A . \cos B + \cos A . \sin B$$

Similarly, the equations $(e^{iA})^2 = e^{i(2A)}$ and so forth yield double angle and triple-angle formulae. Euler's formula makes the derivation of this fact swift and easy.

2.4 GRAPH THEORY

The famous Seven Bridges of Königsberg Puzzle was inspired by an actual place and situation. There was a question about whether it was possible to walk a path that directly crosses each of the seven bridges only once. The seven bridges were located in Königsberg, Prussia, which is now known as Kaliningrad, Russia. No one had managed to prove if it was possible or not, until Euler proved that it was impossible in 1736.

Euler proved the result using graph theory, a study of graphs. First, he eliminated all the features around except the unbroken sections of land and the bridges connecting them. He replaced each part of land with a dot called a vertex, the fundamental unit out of which graphs are formed, or node, a point at which a curve intersects itself, and used lines to represent the bridges. The outcome of this was a mathematical structure known as a graph.

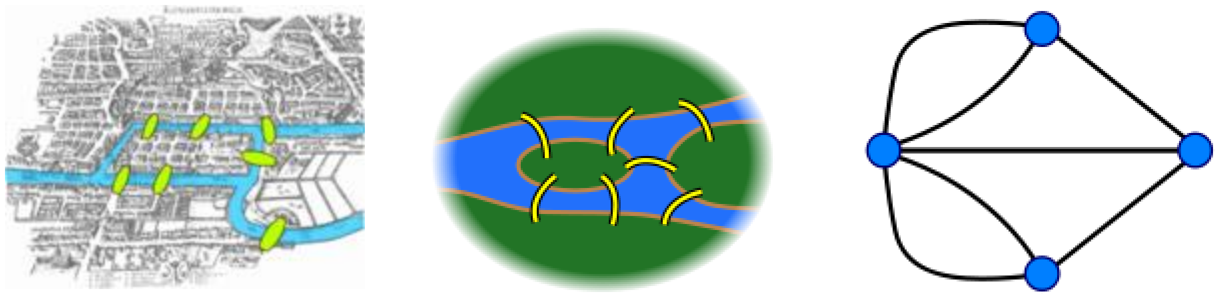

Figure 2.4.1 Euler's Method to Solving the Puzzle

Euler found out that the solution to this problem lay in the degree of the nodes, which is the number of edges touching it. After creating the graph of the bridges, Euler saw that three nodes have degree 3 and one has degree 5. He then proved that a circuit of the entire area was only possible is there were only two or zero nodes of odd degree. Such a path is called a Eulerian path or Euler walk. Also, if there are two nodes with an odd degree, the nodes would be the starting and ending points of an Eulerian path. Because the graph made of Königsberg has four nodes of an odd degree, it cannot have an Eulerian path; so therefore, it was proven impossible to make a route across each of the bridges only once.

Euler's solution of the Bridge problem is considered to be the first theorem of graph theory. Graph theory is now generally considered a part of combinatorics. Euler's realization that the main information was the number of bridges and the list of their endpoints started the development of topology, an extension of geometry. The difference between the actual layout and the graph is an example of the idea that topology is not about the rigid shape of objects.

3. CONCLUSION

From this research, I have learnt more about Euler and have begun to appreciate his contributions and how his discoveries play a great role in modern mathematics. Euler's works were so well-known that many of it was named after him.

Euler's formula is $e^{ix} = cos\ x + i\ sin\ x$ which can be used to find complex numbers; very useful today whereas complex numbers cannot be found using calculators.

Euler's discoveries are very useful throughout all areas of mathematics, bringing mathematics to a higher level.

REFERENCES

1. Wikimedia foundation, Inc., 2001, Biography of Leonhard Euler
   Retrieved March 13,2007, from
   http://en.wikipedia.org/wiki/Leonhard_euler

2. Ask Dr. Math, 1994, Drexel University, Proof for Euler's number
   Retrieved March 15, 2007, from
   http://mathforum.org/dr.math/faq/faq.e.html

3. Wikimedia foundation, Inc., 2001, Euler's Formula
   Retrieved March 15, 2007, from
   http://en.wikipedia.org/wiki/Euler%27s_formula

4. Wikimedia foundation, Inc., 2001, Seven Bridges of Königsberg Puzzle
   Retrieved March 15, 2007, from
   http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg